Extra Dimensions: Small, Large and Universal

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Marcela Carena, Eduardo Ponton and Tim Tait:
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and **Antonio Delgado**: Phys. Rev. D 68, 035010 (2003).

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Extra Dimensions: Large, Small and Universal

- Introduction:

- Extra Dimensions:
  Flat Space and Warped Extra Dimensions

- The hierarchy problem and bulk gravity
- The Warped Extra Dimension Scenario
- Bulk gauge fields
  - Transparent and Opaque Branes
  - Effects of a Localized Higgs Field
  - Fit to the Electroweak Precision Data

- Universal Extra Dimensions
  - Dark Matter
  - Six Dimensions: Chirality, N_f and Proton Stability

Extra Dimensions

- Standard successful description of nature depends crucially on the existence of three spacial dimensions.
- No information about the behavior of gravity in the submilimiter range and of the standard model interactions at energies above the weak scale.
- First attempt to unify the description of gravity and electromagnetic interactions relied on an extra dimension.
- They are required in string theory framework, which, however, provides no guidance about their size.

• They should be compact (small)

 – If seen by SM particles $\longrightarrow$ they should be of subatomic size: R $\leq 10^{-17}$ cm $\approx 1/TeV$

 – If seen only by gravity, $\longrightarrow$ they can be larger: R $\leq 0.1mm$

If gravity propagates in the extra dimensions $\Longrightarrow$
Newton's law modified: $M_{Pl}^2 = (M_{Pl}^{fund.})^{2+d} R^d$

$\Longrightarrow$ This lowers the fundamental Planck scale, depending on the size and number of Extra Dimensions

• Solution to hierarchy problem:
New problem: why R so large?

Warped Extra Dimensions

Possible solution to hierarchy problem $\rightarrow$ warped space:
Non-factorizable metric:
$ds^2 = e^{-2\sigma} \eta_{\mu\nu} dx^\mu dx^\nu + dy^2$,
$\sigma(y) = ky$ $\Longrightarrow$Solution to 5d Einstein eqs.

5d Planck mass relates to M_{Pl}:

$$M_{Pl}^2 = \frac{(M_{Pl}^{\rm fund})^3}{2k} (1 - e^{-2kL})$$

Assuming fundamental scales of the same order:
$M_{Pl} \sim M_{Pl}^{\rm fund} \sim v \sim k$ (k: warp factor)
$\Longrightarrow$ Physical Higgs v.e.v. suppressed by e^{-kL}

$$\tilde{v} = v\, e^{-kL} \simeq M_{Pl}\, e^{-kL}$$

$kL \approx 34 \Longrightarrow$ good solution to the hierachy problem

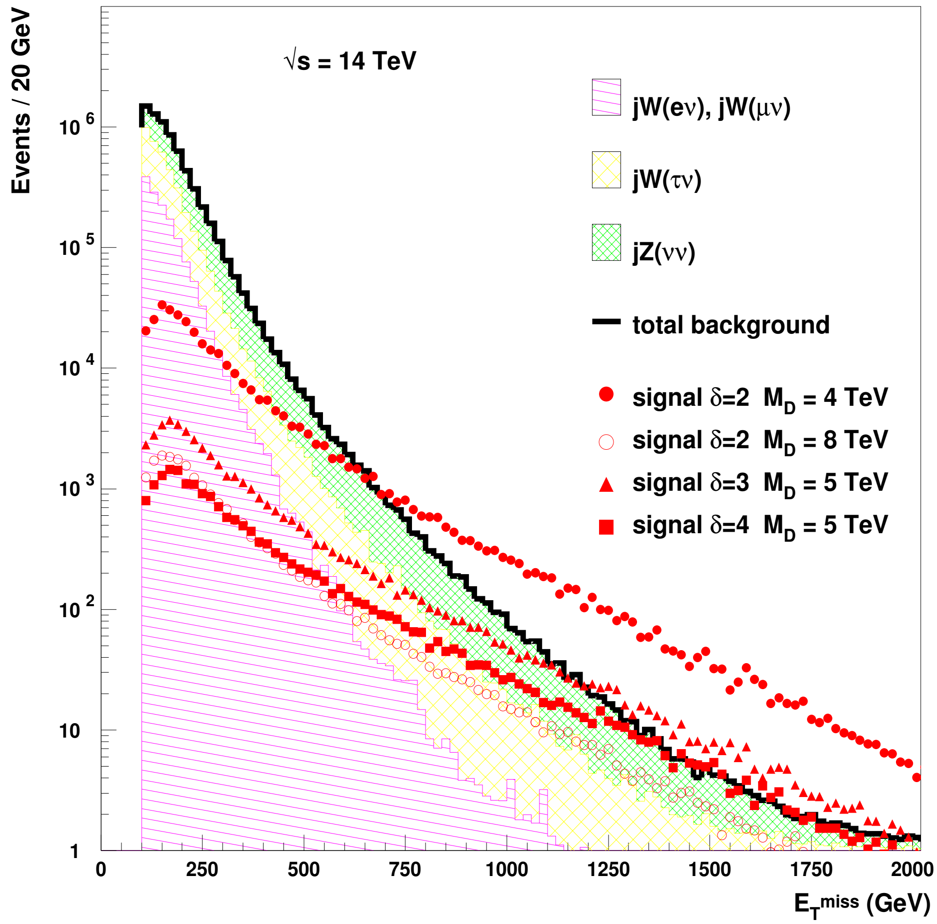
Probing extra dimensions from our 4D world (brane)

4 Dimensional effective Theory
SM particles + gravitons + tower of Kaluza Klein (KK) states with the same quantum number of gravitons
Momentum in extra dimensions is seen as a mass in the four dimensional world : For massless particles
$E^2 - \vec{p}^2 = p_d^2 = m_{KK}^2$, $\qquad m_{KK} = \frac{n}{R}$

Flat case

- Coupling of gravitons to matter has standard $1/M_{Pl}$ strength.
- Number of gravitons accessible at some energy E grows like $N_{KK} = (E \times R)^d$
- $R^{-1} \simeq 10^{-2}$ GeV (d =6); $R^{-1} \simeq 10^{-3}$ eV (d = 2)
- Emission of KK graviton tower states: $G_n \leftrightarrow$ Missing E_T (emitted gravitons appear almost as continuous mass distribution).
- Graviton exchange in $2 \rightarrow 2$ scattering
- Cross section of graviton production grows with the number of accessible gravitons
  $\sigma(f\bar{f} \rightarrow V + \mathrm{Miss.E_T}) \simeq \frac{(E\ R)^d}{M_{Pl}^2} \simeq \frac{E^d}{\left(M_{Pl}^{\mathrm{fund}}\right)^{d+2}}$
  where $V = \gamma, g$ depending on f = l,q.

Discovery reach at the LHC

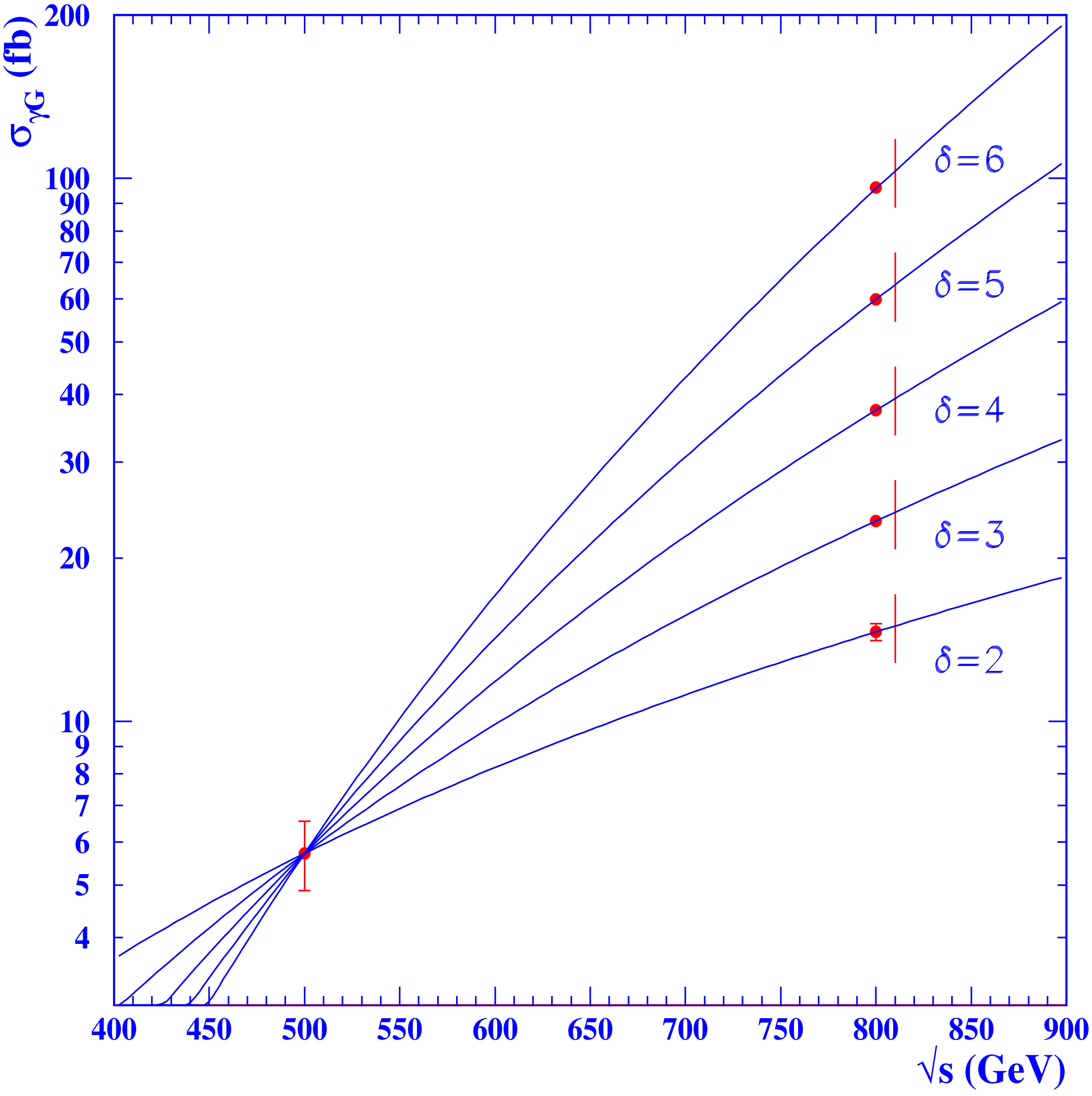


Hinchliffe and Vacavant, '01

Discovery reach depends on E_T^{cut} and the fundamental Planck scale and on the number of dimensions

With 100 fb^{-1}, fundamental scales of 6 TeV and 10 TeV for d = 4,2 can be probed

Determination of number of extra dimensions at a Linear collider

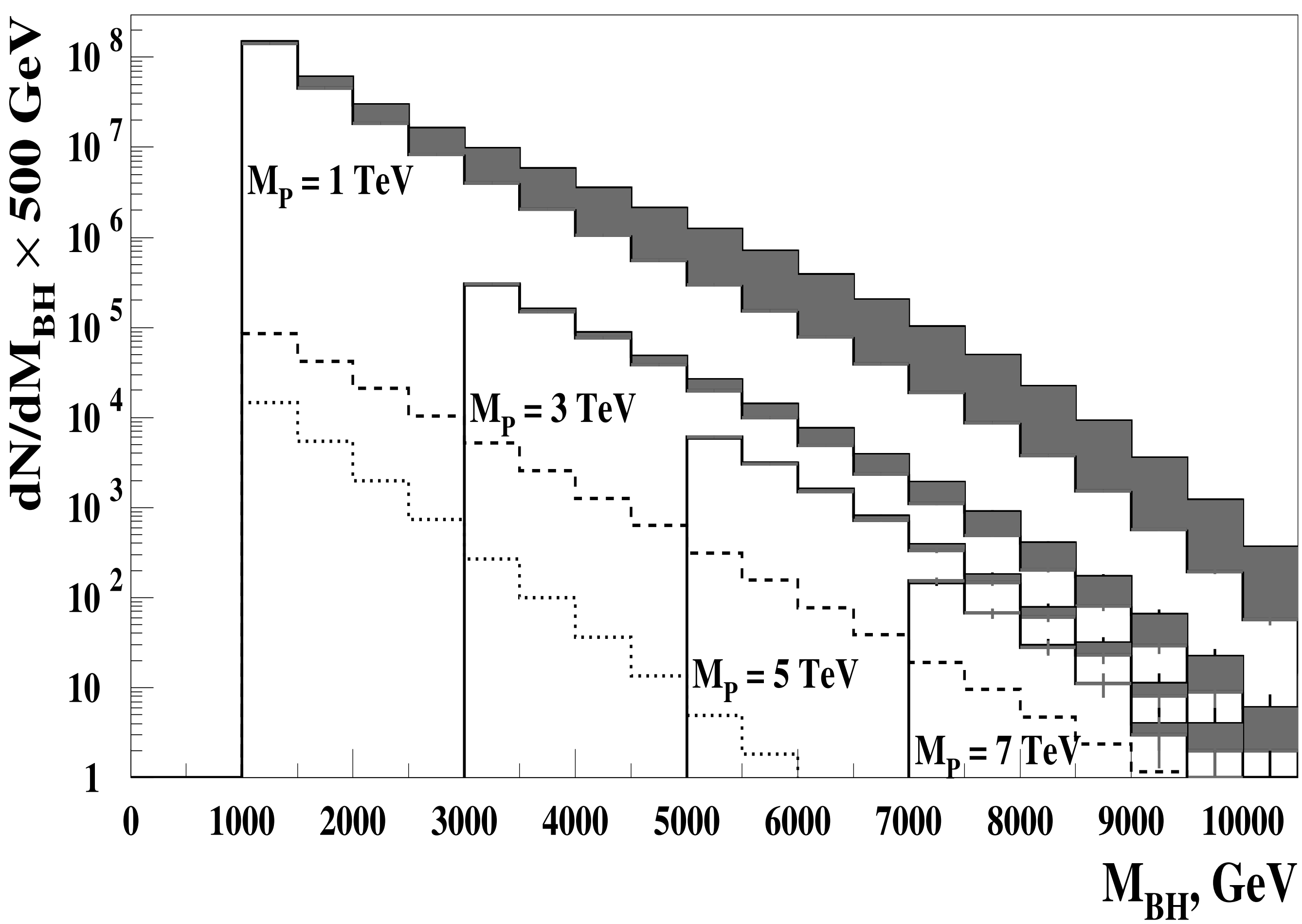


$M_D \simeq 1$ TeV.

Black Hole Production ?

- Two partons with center of mass energy $\sqrt{s} = M_{BH}$, with $M_{BH} > M_{Pl}^{fund}$ collide with a impact parameter that may be smaller than the Schwarzschild radius.
  $R_S \simeq \frac{1}{M_{Pl}^{fund}} \left(\frac{M_{BH}}{M_{Pl}^{fund}} \right)^{\frac{1}{d+1}}$
- Under these conditions, a blackhole may form
- If $M_{Pl}^{fund} \simeq 1$ TeV $\rightarrow$ more than 10^7 BH per year at the LHC (assuming that a black hole will be formed whenever two partons have energies above M_{Pl}).
- Decay dictaded by blackhole radiation, with a temperature of order $1/R_S$. Signal is a spray of SM particles in equal abundances: hard leptons and photons.
- At LHC, limited space for trans-Planckian region and quantum gravity. At a VLHC ($\sqrt{s} \geq 100$ TeV), perfect conditions.

Black Hole production at the LHC



Dimopoulos and Lansberg; Thomas and Giddings '01

Sensitivity up to $M_{Pl}^{\text{fund}} \simeq 5 - 10$ TeV for 100 fb^{-1}.

Astrophysics constraints

- Most relevant constrains come from SN 1987A
- 10^{53} ergs released in a few seconds of SN collapse
- Standard sources account well for this
- Strong constraints on additional sources, like (many) very light gravitons.
- Bounds on fundamental Planck energy depends strongly on astrophysics variables, like the temperature and density of the core
- $T_{\rm core} \simeq 30$–70 MeV

$M_{Pl}^{\rm fund} \gtrsim 50$ TeV for d $= 2$
$M_{Pl}^{\rm fund} \gtrsim 4$ TeV for d $= 3$
$M_{Pl}^{\rm fund} \gtrsim 1$ TeV for d ≥ 4

Cullen, Perelstein

Warped Case

- Graviton KK modes have 1/TeV coupling strength to SM fields and masses starting with a few hundred GeV.
- KK graviton states produced as resonances or may contribute to $f\bar{f}$ production.
- One can rewrite the warp factor and the massive graviton couplings in terms of mass parameters as:

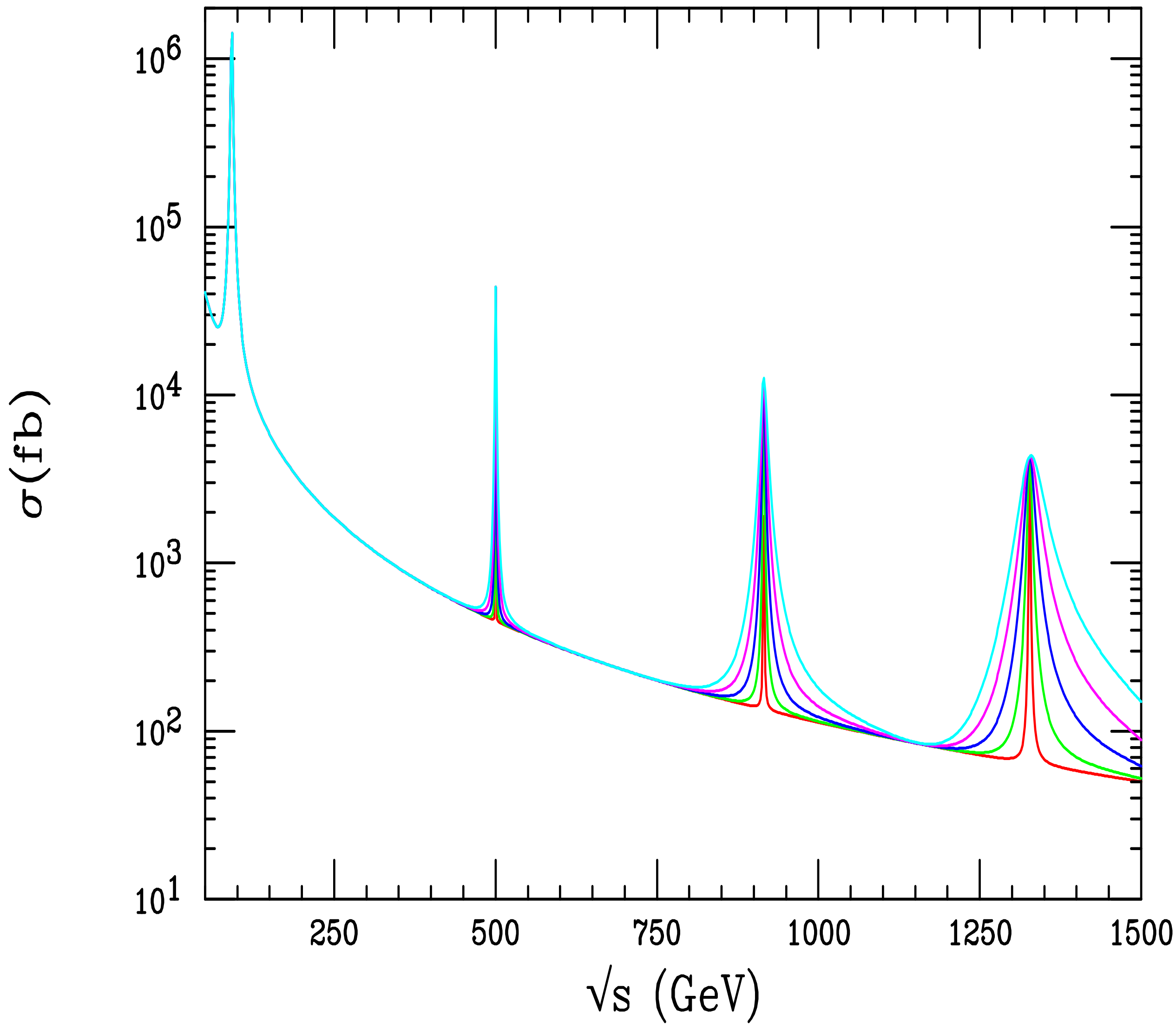
$$\exp(-kL) = \frac{m_n}{kx_n}$$
$$\Lambda_\pi \simeq \frac{\bar{M_{Pl}} m_1}{kx_1} \tag{1}$$

with $x_1 \simeq 3.8$, $x_n \simeq x_1 + (n-1)\pi$.

- Calling $\eta = k/\bar{M_{Pl}}$, one gets that the graviton width is

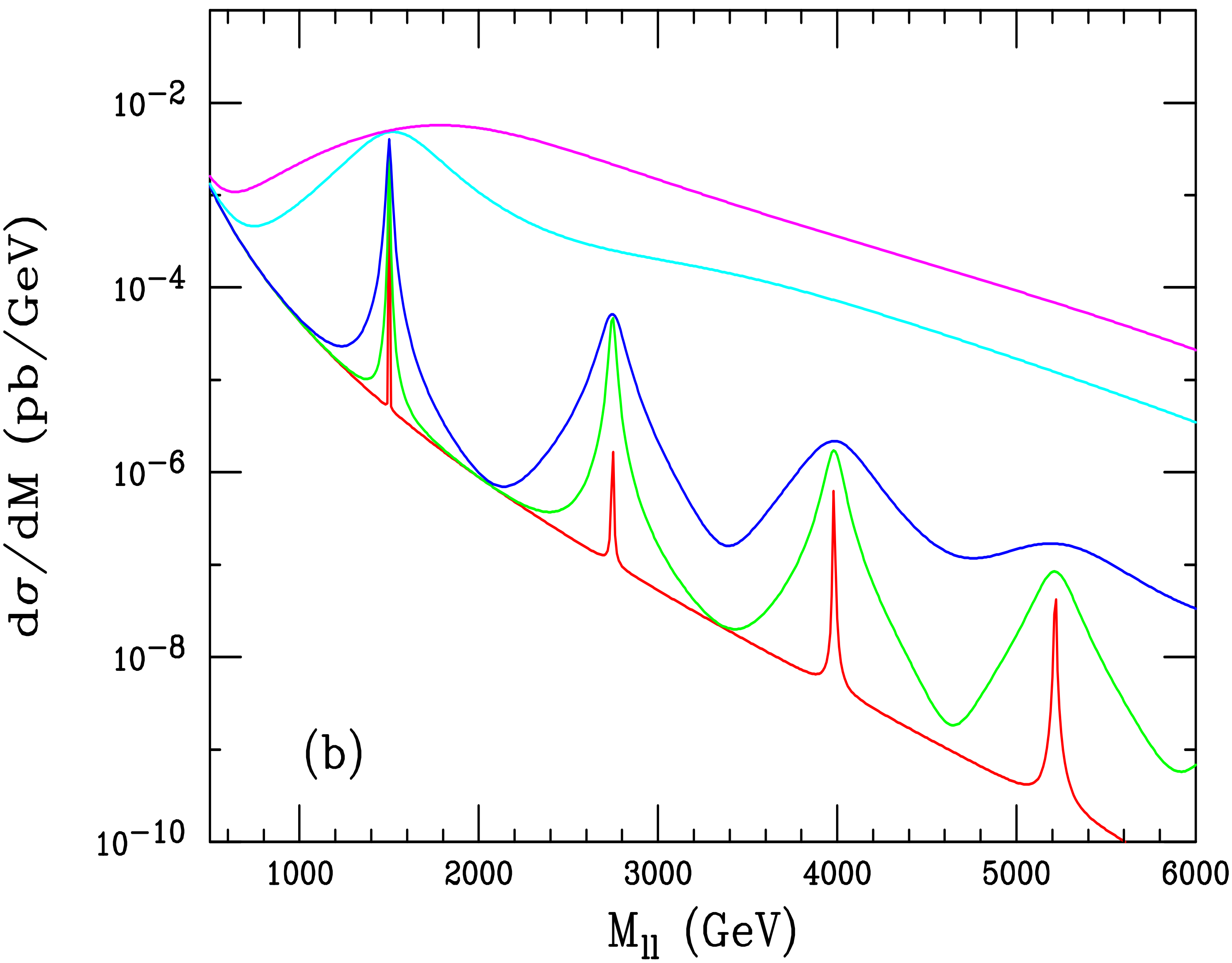
$$\Gamma(G^n) \simeq m_1 \eta^2 \frac{x_n^3}{x_1} \tag{2}$$

Graviton Production at a Linear Collider



$\sigma(e^+e^- \to \mu^+\mu^-)$ as a function of $\sqrt{s}$, including KK graviton exchange, for $m_1 = 500$ GeV and $\eta = 0.01$–0.05

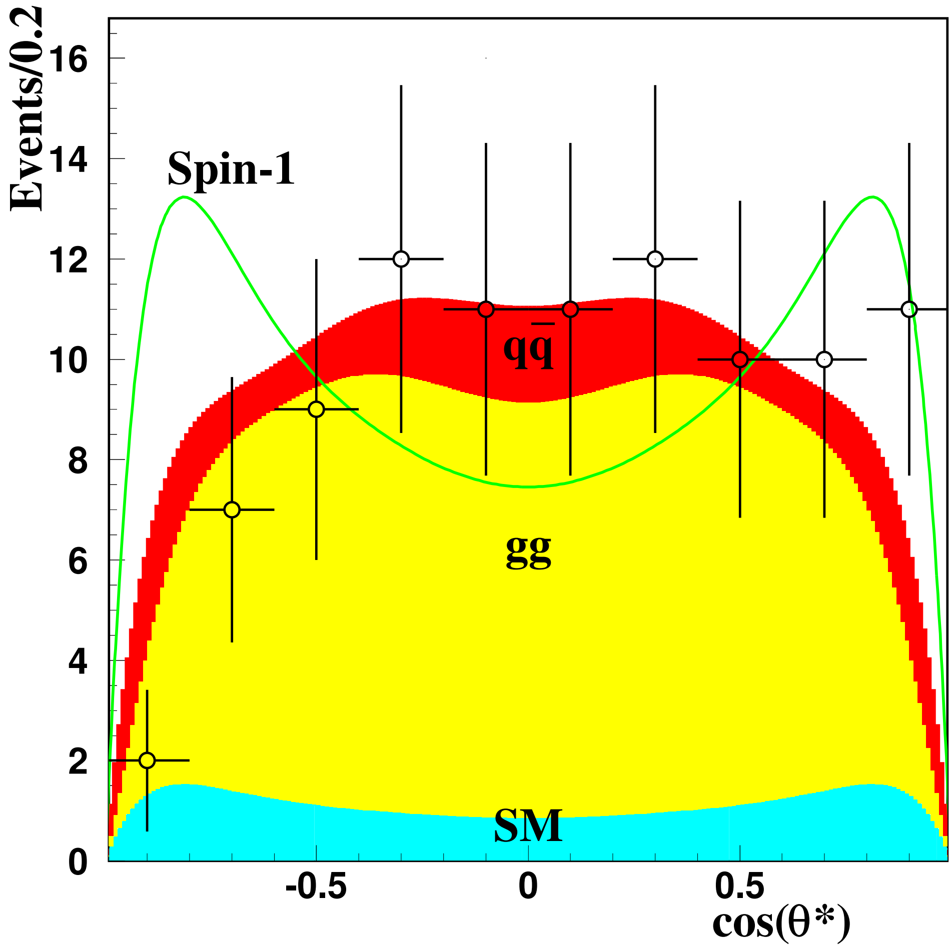
Graviton production at the LHC



Davoudiasl,Hewett and Rizzo'00

From top to bottom $\eta = 1., 0.5, 0.1, 0.05, 0.01$

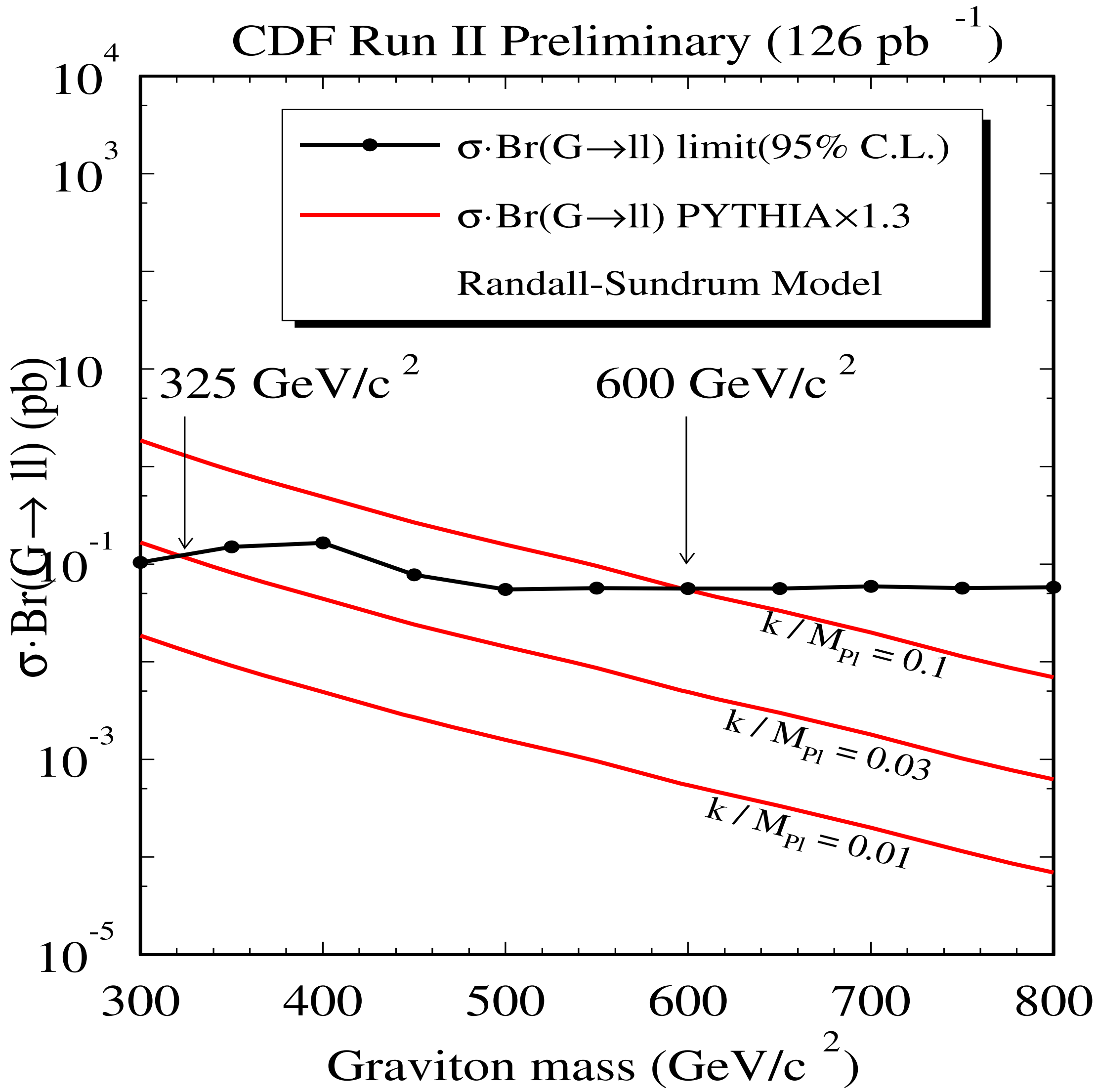
Determination of graviton spin at LHC



Allanach et al. '02

- Use angular distributions to determine the spin of the resonance.
- Spin can be determined with 90% C.L. for $M_G \simeq 1.7$ TeV, with 100 fb^{-1} of luminosity (One can probe narrow gravitons, with widths smaller than experimental resolution, up to 2.1 TeV.).

Graviton Production at the Tevatron collider



$p\bar{p} \to l^+l^-$, with $l = e$ and μ. The Tevatron can already put (rather weak) limits on RS gravitons. With 2 fb^{-1} one will be able to explore the TeV region.

Gauge Fields in the Bulk

• Existence of TeV size or warped extra-dimensions, with gauge fields propagating in the bulk
$\Longrightarrow$ interesting theoretical possibility
Consider a 5d space with fermion fields localized in 3-branes at y =0,L:

$$\mathcal{L} = \frac{-\sqrt{-g}}{4 g_5^2} \left[\mathcal{F}^{MN} \mathcal{F}_{MN} + r_a \mathcal{F}^{\mu\nu} \mathcal{F}_{\mu\nu} \delta(y) + r_b \mathcal{F}^{\mu\nu} \mathcal{F}_{\mu\nu} \delta(y - L) \right]$$

$g \rightarrow$ determinant of the metric. $\qquad \sqrt{-g} = e^{-4ky}$
$M = 0, 1, 2, 3, 5; \qquad \mu = 0, 1, 2, 3 \qquad g_5^{-2} \rightarrow$ dim. of mass,
local brane term coefficients: $r_i = g_5^2 / g_i^2 \rightarrow$ dim. of length
and

$$\mathcal{F}^a_{MN} = \partial_M \mathcal{A}^a_N - \partial_N \mathcal{A}^a_M + f^{abc} \mathcal{A}^b_M \mathcal{A}^c_N$$

Warped extra Dimensions

Allowing for opacity in both IR and UV branes, the orthonormality cond. for the KK mode decomposition becomes $A^{\lambda}(x_{\mu}, y \equiv x_5) = \sum_n f_n(y) A_n^{\lambda}(x^{\mu})$

$$\frac{1}{g_5^2} \int_0^L dy \left[1 + 2\, r_{UV} \delta(y) + 2\, r_{IR} \delta(y - L)\right] f_n(y) f_m(y) = \delta_{nm}$$

$$\frac{1}{g_5^2} \int_0^L dy\, e^{-2ky}\, f_n'(y) f_m'(y) = m_n^2 \delta_{nm}$$

Conditions simultaneously solved if $f_n(y)$ satisfies

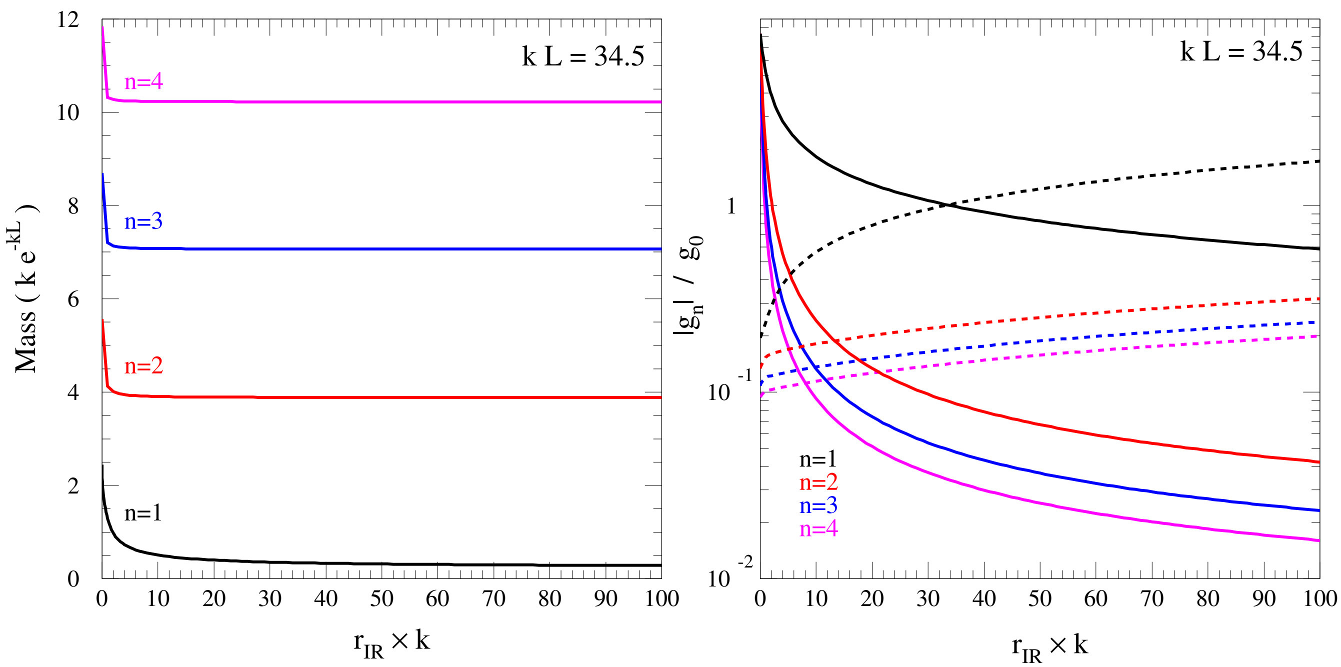
$$\left[\partial_y^2 - 2k\partial_y + e^{2ky} m_n^2 (1 + 2\, r_{UV}\, \delta(y) + 2\, r_{IR} \delta(y - L))\right] f_n(y) = 0$$

(same eq. obtained from imposing that gauge fields are on the mass shell). • The bulk solution for KK modes remains the same after introducing brane opacity • boundary conditions at y=0 and y=L should reflect discontinuity in the derivatives

$$g_0 = f_0(y) = \frac{g_5}{\sqrt{L + r_{IR} + r_{UV}}}$$

Opaque IR Brane

- For sufficiently large kr_{IR} there is a mode with mass of order $k\, e^{-kL}$ that couples to the brane with strength $g_1 = \sqrt{\frac{L}{r_{IR}}}\, g_0$ while the other modes decouple.

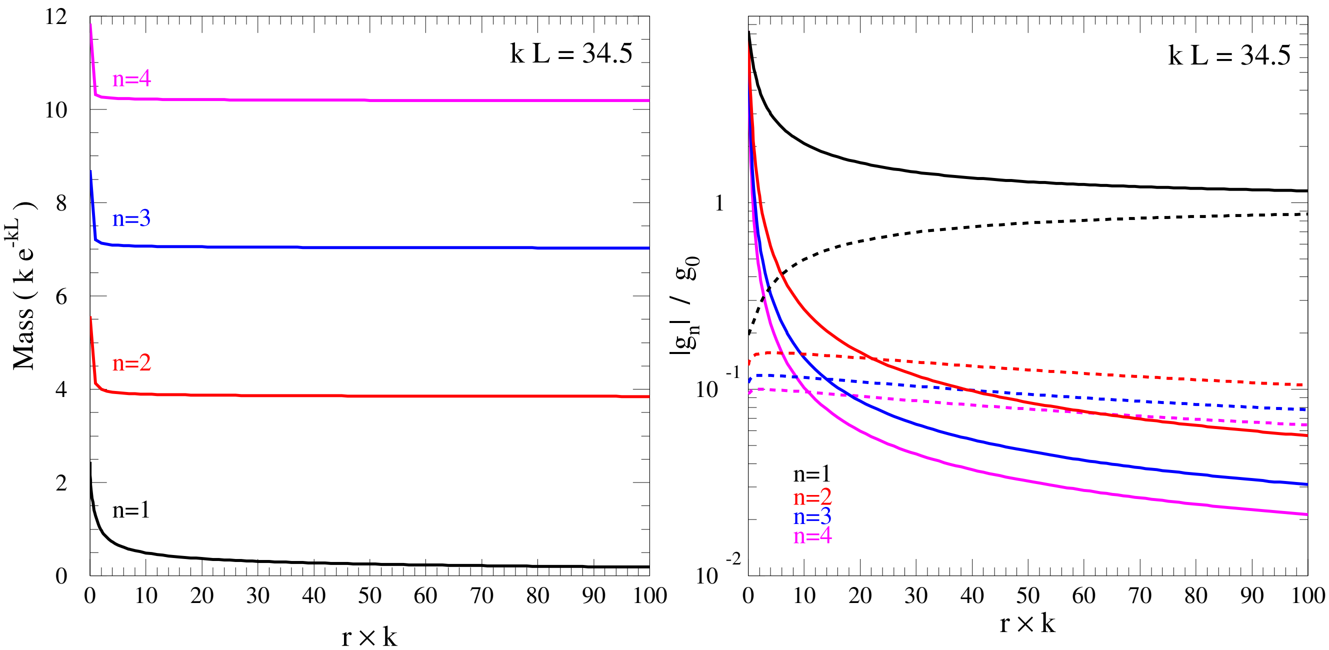


- For $r_{IR} \gg L \Longrightarrow g_1 \ll g_0$: Only the zero mode couples to IR brane matter with $\Longrightarrow g_0^2 \approx g_{IR}^2 = g_5^2/r_{IR}$

Opaque IR and UV Branes

$r_{IR}, r_{UV} \to \infty \longrightarrow$ an observer on either brane must be insensitive to the extra dimensions + other brane.

$\Longrightarrow$ two massless modes should appear, one l.c. of them couples to each brane with local brane coupling strength



$r_{IR} = r_{UV} = r$:

- large r: first KK-mode mass $\to 0$ and its coupling becomes equal (and opposite in sign for UV brane fields) to the zero mode one

Localized Higgs Effect

To address the hierarchy problem, the Higgs responsible for EWSB must be localized on the IR brane

In the low energy effective theory after canonical normalization of the Higgs kinetic term

$$-\int d^4x \left\{ \eta^{\mu\nu} (D_\mu H)^\dagger D_\nu H + \lambda \left(|H|^2 - \frac{1}{2} v^2 e^{-2kL} \right)^2 \right\}$$

yielding a localized gauge boson mass prop. to the localized v.e.v., $\tilde{v} = e^{-kL} v$

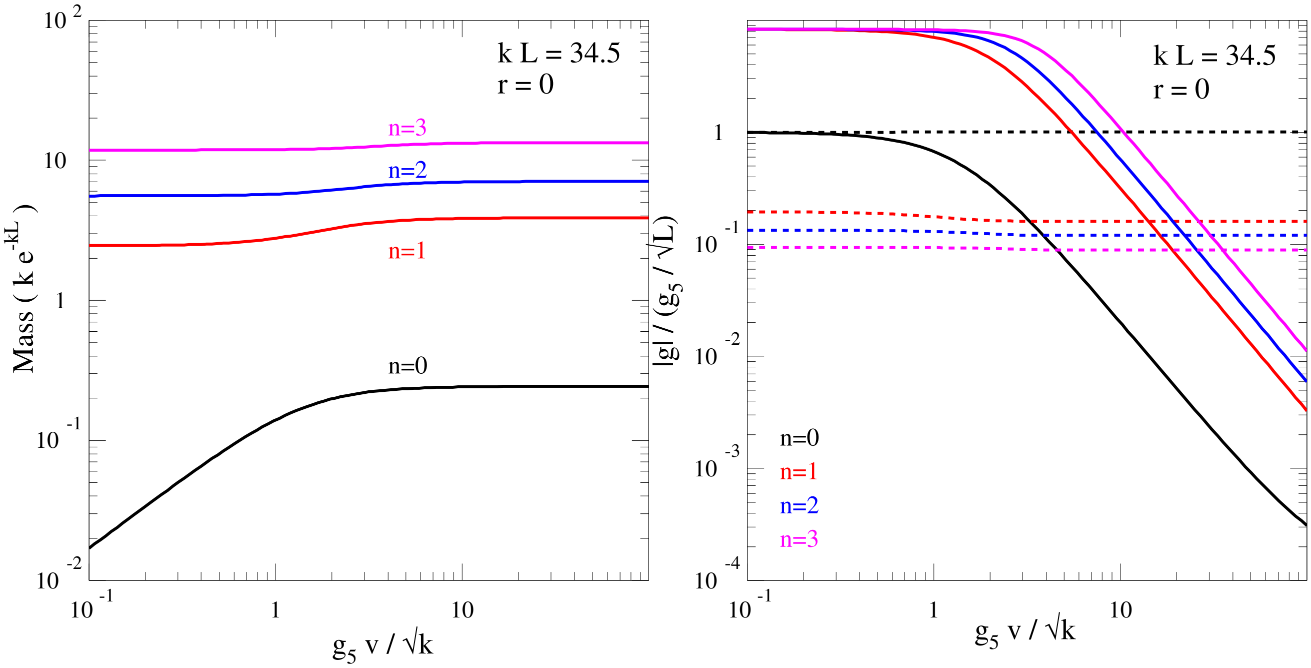
Effects on the gauge field propagation induced by a Higgs v.e.v. on the IR brane The presence of the v.e.v. tries to induce a repulsion of the gauge field from the brane location

For $v \gg k$

• Zero mode no longer flat with mass more and more insensitive to v as it bends away from the brane.

• KK mode gauge couplings with $m_n/ke^{-kL} > g_5 v/\sqrt{k}$, and zero mode coupling, tend to small values.

• The ratio of the KK mode couplings to the zero mode tends to a constant larger than $\sqrt{2\,k\,L}$ $\quad$ ($v = 0$ value)



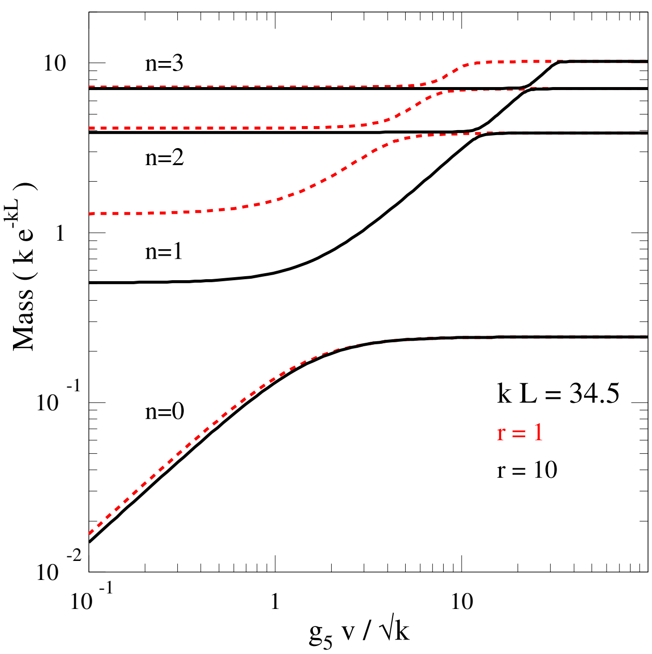
k L = 34.5
r = 0
n=3
n=2
n=1
n=0
Mass (k e^-kL)
g_5 v / √k
k L = 34.5
r = 0
n=0
n=1
n=2
n=3
|g| / (g_5 / √L)
g_5 v / √k

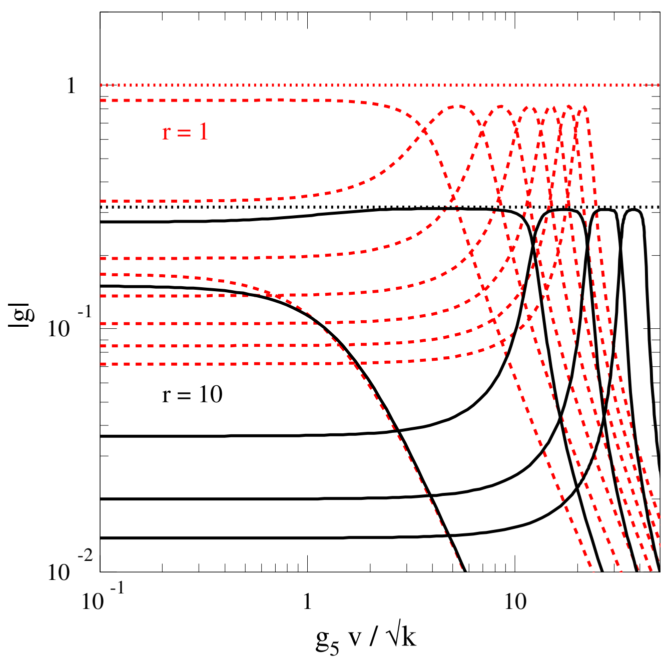
Higgs Field in Opaque IR Brane

From the propagator with endpoints at the IR brane

$$G_p(L,L) \sim -g_5^2/\left(p^2\, r_{IR} + g_5^2 \tilde{v}^2\right) \qquad p \gg k\, e^{-kL} \quad (r_{IR} k > 1)$$

$$G_p(L,L) \sim -g_5^2/\left[p^2\,(L + r_{IR}) + g_5^2 \tilde{v}^2\right] \qquad p \ll k\, e^{-kL}$$





- For $v \ll k$ and $p \ll k\, e^{-kL}$, $\rightarrow$ single state with $g_0 \simeq g_5/\sqrt{L + r_{IR}}$ and $m_0 \simeq g_0 \tilde{v}$, up to correc. of order $g_5^2 v^2/k \Longrightarrow$ zero mode grows linearly with $\tilde{v}$

- If $r_{IR} \gg L$, both propagators describe a single 4d state with mass $g_{IR}\,\tilde{v}$ and coupling g_{IR}.
All other modes decouple.

- For $v/k \gtrsim 1 \rightarrow$ zero mode mass becomes insensitive to v; other modes successively have masses prop. to v. and couplings $g_{IR} = g_5/\sqrt{r_{IR}}$

Electroweak Theory

For a common term r for U(1) and SU(2), and going to the basis $\mathcal{W}^3_\mu = c^2 \mathcal{Z}_\mu + \mathcal{A}_\mu \quad \mathcal{B}_\mu = -s^2 \mathcal{Z}_\mu + \mathcal{A}_\mu$, $s \equiv g'_5/\sqrt{g_5^2 + {g'_5}^2}$.

$$\mathcal{L}^5_{EW} = \sqrt{-g}\left\{-\frac{s^2}{2e_5^2}\mathcal{W}^+_{MN}\mathcal{W}^{MN}_-\left[1+2r\delta(y-L)\right] - \frac{1}{4e_5^2}\mathcal{F}_{MN}\mathcal{F}^{MN}\left[1+2r\delta(y-L)\right] - \frac{s^2c^2}{4e_5^2}\mathcal{Z}_{MN}\mathcal{Z}^{MN}\left[1+2r\delta(y-L)\right] - 2v^2\delta(y-L)\left(\mathcal{W}^+_M\mathcal{W}^M_- + \frac{1}{2}\mathcal{Z}_M\mathcal{Z}^M\right)\right\}$$

with $1/e_5^2 = 1/g_5^2 + 1/{g'_5}^2 \rightarrow$ 5d photon coupling.

- The quantities m_W and m_Z are given by

$$m_Z = \frac{e\tilde{v}}{s\,c}\left(1 - \eta\,\epsilon + \cdots\right) \qquad m_W = \frac{e\tilde{v}}{s}\left(1 - c^2\eta\,\epsilon + \cdots\right)$$

where

$$\eta\,\epsilon = \frac{2k^2L^2 - 2kL + 1}{8k(L + r_{IR})}\,\frac{e^2v^2}{s^2c^2k^2}\ .$$

– The Z zero mode mass, $m_Z \equiv M_Z = \frac{e\tilde{v}}{s_0\,c_0}$

This defines s as a function of s_0

$$s = s_0\left(1 - \frac{c_0^2}{c_0^2 - s_0^2}\eta\,\epsilon + \cdots\right)$$

Fermi constant

- f_W, f_Z and $f_A \to$ zero mode wave functions at $y = L$,

$f_A = e_5/\sqrt{L+r} \equiv e$

$f_Z = \frac{\sqrt{g_5'^2+g_5^2}}{\sqrt{L+r}}(1-2\eta\epsilon) \equiv (e/sc)\hat{f}_Z = \sqrt{g^2+g'^2}\hat{f}_Z$

$f_W = \frac{g_5}{\sqrt{L+r}}(1-2c^2\eta\epsilon) \equiv (e/s)\hat{f}_W = g\hat{f}_W$ The photon experiences no symmetry-breaking

$\to$ zero mode wave function is flat

Use precisely measured quantities: α_Z, M_Z, G_μ to determine SM electroweak parameters e, $\sin^2\theta_W \equiv s_0$, and $\tilde{v}$.

Determine e_5, s and v from data and leave r, L as free parameters in the fit

- α_Z^{-1}. yields $e = e_5/\sqrt{L+r}$
- W boson at zero momentum transfer with entire KK tower effects:

$1/\tilde{v}^2 = 4\sqrt{2}G_\mu \equiv f_W^2/m_W^2 + \sum_{n\neq 0} f_{W_n}^2/m_{W_n}^2$

with $G_\mu = 1.16639(1) \times 10^{-5}$ GeV^{-2} yields $\tilde{v} \simeq 123$ GeV. Fit to precision observables $\Longrightarrow$ determine r and L region consistent with data.

Parametrization in terms of Oblique Corrections

Once G_μ, M_Z and α_Z are fixed, all corrections to the zero mode tree-level Lagrangian may be absorbed into corrections to the gauge bosons wave functions, and the W-mass.

$\Longrightarrow$ always possible when fermion couplings corrections are flavor independent $\Longrightarrow$ rescaling fermion interactions to unity redefining the gauge boson fields.

Therefore, although the corrections are at tree-level, they can be parametrized as a function of the same oblique parameters as the one-loop corrections

Tree-level 5d contributions to S, T and U

$$\begin{aligned}
\overline{S} &= \frac{4s^2c^2}{\alpha}\left(1-\frac{1}{\hat{f}_Z^2}\right) = -16 s_0^2 c_0^2 \frac{\eta\epsilon}{\alpha} \\
\overline{T} &= \frac{1}{\alpha}\left(\frac{m_W^2}{c^2 m_Z^2 \hat{f}_W^2} - \frac{1}{\hat{f}_Z^2}\right) = -2 s_0^2 \frac{\eta\epsilon}{\alpha} \\
\overline{U} &= \frac{4s^2}{\alpha}\left[1-\frac{1}{\hat{f}_W^2} - c^2\left(1-\frac{1}{\hat{f}_Z^2}\right)\right] = \mathcal{O}(\epsilon^2)
\end{aligned}$$

Non-Oblique Corrections

Due to non-oblique corrections associated with heavy KK modes $\rightarrow$ define effective S, T and U parameters to describe the precision electroweak observables.

• Z-pole precision observables properly described introducing an effective parameter, $T_{\rm eff}$, which includes the non-oblique corrections to the weak mixing angle,

$$s^2 - s_0^2 = \frac{\alpha}{c^2 - s^2}\left(\frac{1}{4}S - s^2 c^2 T_{\rm eff}\right) , \qquad (3)$$

where $T_{\rm eff} = T + \Delta T$

$$\Delta T = -\frac{1}{\alpha}\frac{\delta G_\mu}{G_\mu} = -\frac{2c_0^2}{\alpha}\eta\,\epsilon + \mathcal{O}(\epsilon^2)$$

$\delta G_\mu \rightarrow$ non-oblique contribution from KK mode exchange. This definition of $T_{\rm eff}$ recovers the relation between s and s_0 derived from M_z

• The non-oblique corrections to G_μ affect also the expression of m_W/m_Z, but with a relative coefficient between the oblique and non-oblique corrections encoded in $T_{\rm eff}$ different from the one in s^2.

To properly parameterize m_W^2/m_Z^2 in terms of effective parameters $S_{\rm eff} = S$, $T_{\rm eff}$ and $U_{\rm eff}$

$$\frac{m_W^2}{m_Z^2} - c_0^2 = \frac{\alpha c^2}{c^2 - s^2}\left(-\frac{1}{2}S_{\rm eff} + c^2 T_{\rm eff} + \frac{c^2 - s^2}{4s^2}U_{\rm eff}\right) ,$$

one can introduce $U_{\rm eff} = U - 4s^2\Delta T$.

Effective S, T, U parameterization

serves to describe all Z-pole observables as well as m_W.

The tree-level five-dimensional contributions, including the non-oblique corrections to G_μ give:

$$\overline{S}_{\rm eff} = -\frac{16 s_0^2 c_0^2}{\alpha}\eta\,\epsilon + \ldots \approx -366\,\eta\,\epsilon\ ,$$

$$\overline{T}_{\rm eff} = -\frac{2}{\alpha}\eta\,\epsilon + \ldots \approx -258\,\eta\,\epsilon\ ,$$

$$\overline{U}_{\rm eff} = \frac{8 s_0^2 c_0^2}{\alpha}\eta\,\epsilon + \ldots \approx 183\,\eta\,\epsilon\ ,$$

We see that $\overline{S}_{\rm eff}$ and $\overline{T}_{\rm eff}$ are negative while $\overline{U}_{\rm eff}$ is positive (and not small).

The full $S_{\rm eff}$, $T_{\rm eff}$, and $U_{\rm eff}$ is given as the sum of the extra dimensional contributions, and the Higgs contributions

$$S_H \simeq \frac{1}{12\pi}\log\left(\frac{m_h^2}{m_{ref}^2}\right)$$

$$T_H \simeq -\frac{3}{16\pi c_0^2}\log\left(\frac{m_h^2}{m_{ref}^2}\right)$$

$$U_H \simeq 0$$

Comparison with Data

We consider two different fits to the precision electroweak data: (Altarelli et al.)

- based on the analysis of the whole available SLD/LEP and Tevatron data

$$
\begin{aligned}
S &= 0.00 \pm 0.11 \\
T &= -0.03 \pm 0.13 \\
U &= 0.27 \pm 0.14 \ .
\end{aligned} \tag{4}
$$

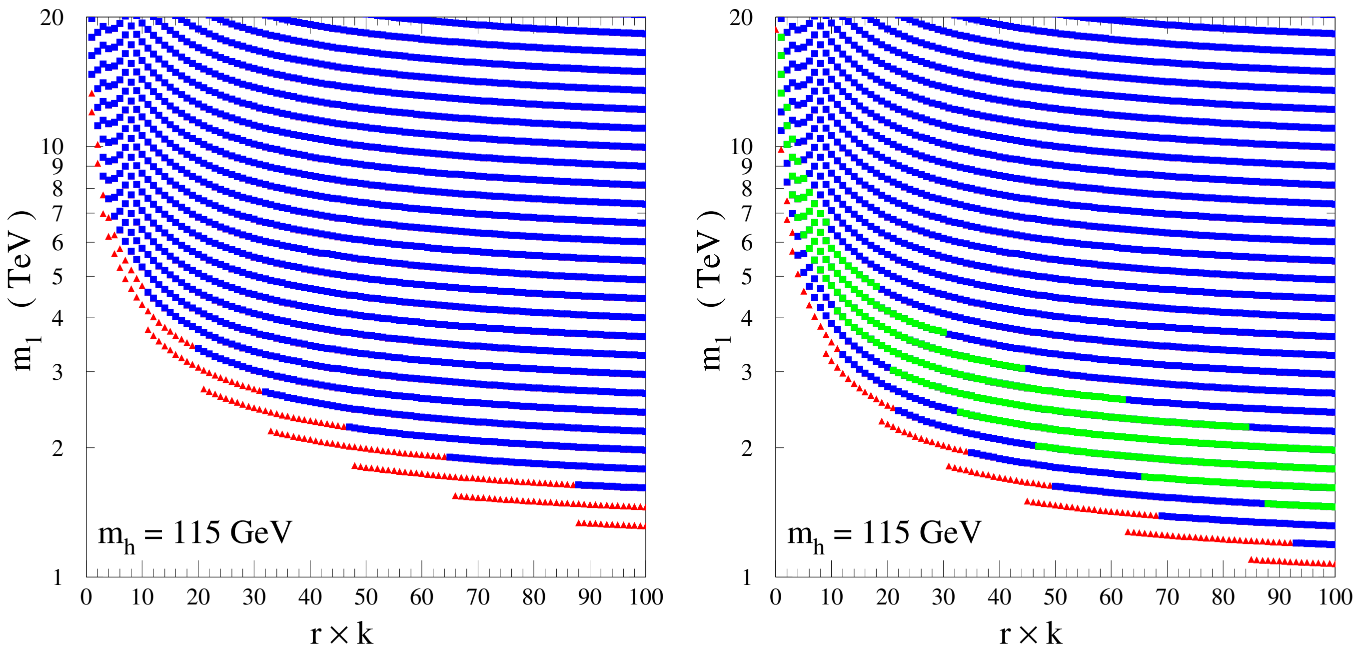
- ignoring the b-forward backward asymmetry, A^b_{FB},

$$
\begin{aligned}
S &= -0.14 \pm 0.12 \\
T &= -0.08 \pm 0.13 \\
U &= 0.20 \pm 0.14 \ ,
\end{aligned} \tag{5}
$$

Note: weak mixing angle extracted from lepton asymmetries differs by more than 3 σ from its value from the hadron asymmetries.

m_H from the EW fit close to its experimental limit only by virtue of A^b_{FB} (measured A^b_{FB}, about 2.6 away from SM prediction). Removing the hadron asymm. from the fit $\rightarrow$ considerably lower values of m_H.

Fit to precision Data



M. Carena, E. Ponton, T. Tait and C.E.M. Wagner

Green: Improvement over SM; Red: Three sigma agreement

- For $r \to 0 \Longrightarrow m_1 \gtrsim 20$ TeV.

- As r increases $\Longrightarrow$ a first KK photon with fermionic couplings of the order of the zero mode couplings, and a mass of a few TeV may appear

- For $r \times k \sim 20$ and $m_1 \sim 3.5$ TeV $\Longrightarrow$ fit to the entire data set as good as in the SM, and the fit to the data without A^b_{FB} consistent within 1σ (improvement over SM fit!)

- Varying $m_h = 200$ GeV $\to$ small variation

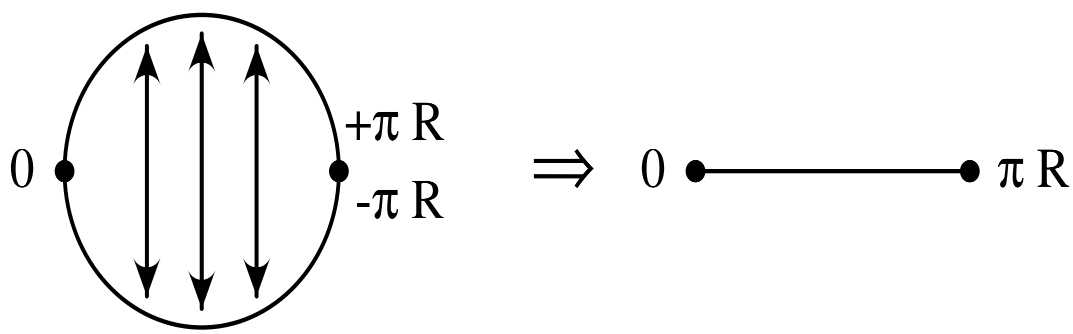
Universal Extra Dimensions

Most natural extension of four dimensional description:

- All particles live in all dimensions, including quarks, leptons, Higgs bosons, gauge bosons and gravitons.
- Universality implies a translational invariance along the extra dimension, and thus conservation of the component of momentum in the that direction.
- This implies that a KK state with $n \neq 0$, carrying non-zero momentum in the extra dimension, cannot decay into standard, zero modes.
- The lightest KK particle is stable, being a good dark matter candidate.
- Other intersting properties that arise in six dimensions are natural proton stability and an explanation of the number of generations

Orbifold

- Massless 5d spinors have 4 components, leading to mirror fermions at low energies.
- If extra dimension is compactified in a circle, no standard chiral theory may be obtained.
- Chiral theories may be obtained by invoking orbifold boundary conditions, projecting out unwanted degrees of freedom.
- Fold the extra dimension, identifying y with $-y$



- Boundary Conditions:
  $\Psi(-y) = \gamma_5 \Psi(y)$
  $V_\mu(-y) = V_\mu(y), V_5(-y) = -V_5(y)$

KK Decomposition

- We expand fields in KK modes:

$$\Phi(x^\mu, y) = \sum_n f^n(y)\Phi^n(x^\mu) \tag{6}$$

- Flat, universal extra dimension:
  - Even fields (A_μ, Ψ_L) have zero modes:

$$\Phi(x^\mu, y) = \sqrt{\frac{1}{\pi R}}\Phi^0(x^\mu) + \sum_{n\geq 1}\sqrt{\frac{2}{\pi R}}\cos\left(\frac{ny}{R}\right)\Phi^n(x^\mu) \tag{7}$$

  - Odd fields (A_5, Ψ_R) don't:

$$\Phi(x^\mu, y) = \sum_{n\geq 1}\sqrt{\frac{2}{\pi R}}\sin\left(\frac{ny}{R}\right)\Phi^n(x^\mu) \tag{8}$$

  - KK masses (before EWSB): n/R

- KK fermions are Dirac, with vector-like interactions.

- In a chiral theory, the left- and right-handed zero modes each have a *separate* tower of KK modes.

- This is somewhat like SUSY, with each SM particle accompanied by partner fields.

KK Parity

- Conservation of KK number is broken to conservation of KK parity: $(-1)^n$.
- KK-parity requires odd KK modes to couple in pairs:
- The lightest first-level KK mode is stable.
- First level KK modes must be pair-produced.
- The Lightest Kaluza-Klein Particle plays a crucial role in phenomenology, similar to the LSP of SUSY:
- All relic KK particles decay to LKPs.
- Any first level KK particle produced in a collider decays to zero modes and an LKP.
- KK parity is also present with boundary fields, provided the same fields live on both boundaries.

Identity of the LKP

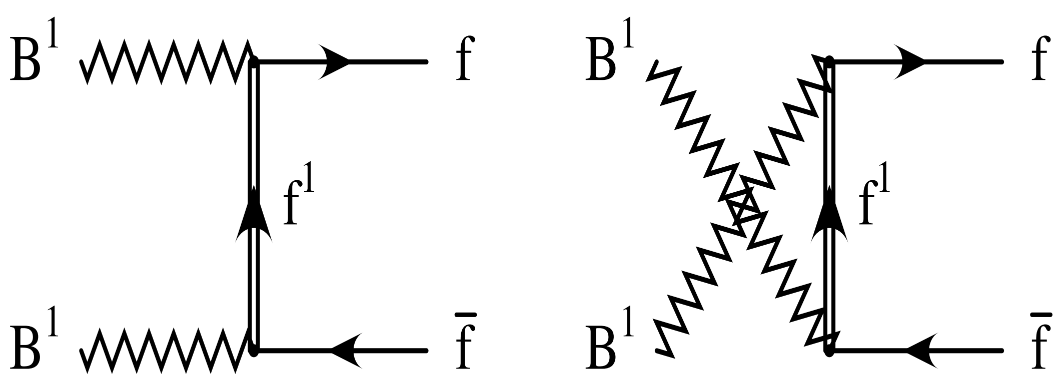
- Boundary terms play a role analogous to soft SUSY-breaking masses, determining masses and couplings for the entire KK tower.
- If we imagine the boundary terms are zero at the cut-off scale, they will be induced at loop size:

$$\delta M^2 \quad \sim \quad \frac{1}{R^2}\frac{\alpha}{4\pi}\log\left(\Lambda R\right) \tag{9}$$

- This prescription is kind of like specializing to mSUGRA from the MSSM.
- Since $\alpha_1 \ll \alpha_2 \ll \alpha_3$, we can imagine that the smallest corrections are to the U(1) gauge boson:
- Since $\delta M \sim 1/R \gg M_W$, the LKP is (almost) purely a KK mode of the U(1) gauge boson: $B_\mu^{(1)}$.
- Following this line of reasoning, the NLKP is the right-handed electron: $e_R^{(1)}$.

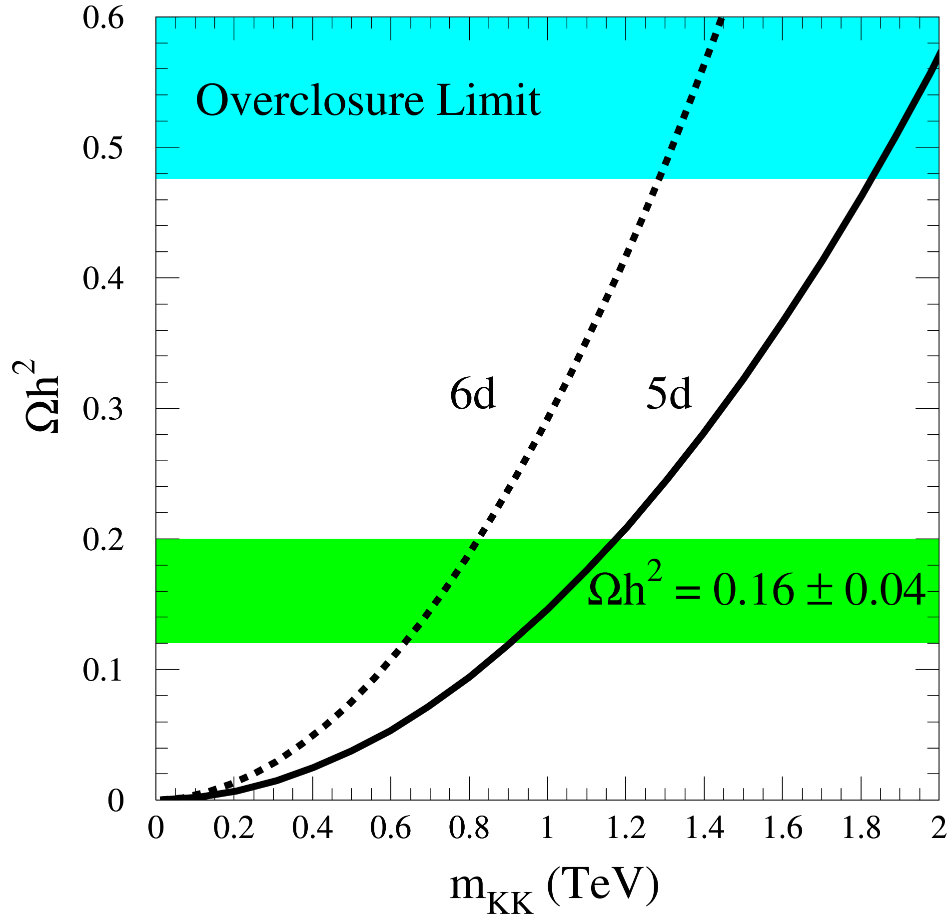
Dark Matter

- Relic Density depends strongly on annihilation cross section.
- In the case of universal extra dimensions, dominant annihilation diagram is given by interchange of first tower of KK particles.



- Whenever the KK mode of the right-handed leptons is close enough in mass to the LKP, coannihilation should be also taken into account

Relic Density: Results

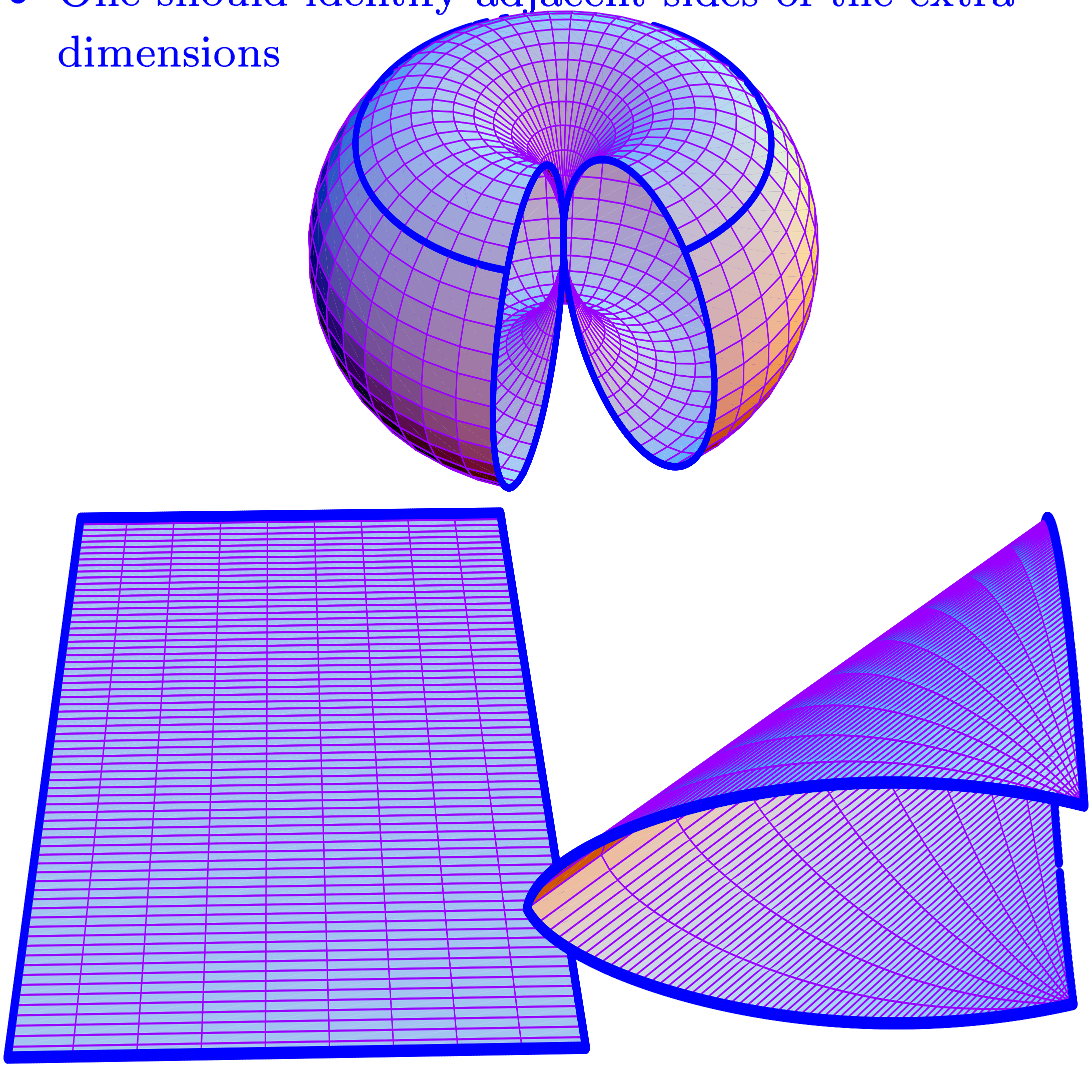


G. Servant, T. Tait '02

Universal extra dimensions of the order of 500 GeV–1 TeV preferred for the LKP to be a good dark matter candidate

Two Universal extra Dimensions

- As happens in the case of one extra dimensions, compactification in a circle (torus in this case) does not lead to chiral fermions
- One should identify adjacent sides of the extra dimensions



Magic of 6 Dimensions

- Cancellations of global gauge anomalies demands that the number of generations $N = 0$ mod 3
- 6d Gauge and Lorentz invariance also demand that all operators fulfill the condition

$$3\Delta B + \Delta L = 0 \tag{10}$$

Proton stability

- In the SM, proton is stable.
- Simplest operators lead to the decay of

$$p \to e^+\pi^0, \qquad \text{withstrength} \simeq \left(\frac{m_p}{M}\right)^4 \tag{11}$$

This leads to a bound on $M \geq 10^{16}$ GeV

- In six dimensions, the constraint on ΔB and ΔL specified above, implies that the dominant decay is $p \to e^-\nu_e\nu_\mu\pi^+\pi^+$
with strength $\simeq \left(\frac{1}{MR}\right)^{12}\left(\frac{m_P}{M}\right)^{10}$
Consistent with experimental bounds even if $M \simeq$ few TeV.

Appelquist, Dobrescu, Ponton and Yee

Conclusions

- Extra Dimensions may be used to address some of the fundamental problems in particle physics
- They lead to new solutions to the hierarchy problem
- In certain frameworks, they may lead to novel answers to (in)frequently asked questions, like the number of families and the proton stability
- They can also have an impact on cosmology, by leading to new dark matter possibilities and perhaps to a new framework to address the cosmological constant problem
- They also give new answers to the problem of flavor and the smallness of neutrino masses, and have an impact on unified scenarios.
- Most important, they may be tested by current or future collider data.